MAGNETIC-MOMENT EVIDENCE FOR THE POLYSPHERON STRUCTURE OF THE LIGHTER ATOMIC NUCLEI*

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In the polyspheron theory of nuclear structure,[1] approximate wave functions for the normal state and some excited states of nuclei are constructed of localized $1s$ orbitals occupied by one to four nucleons; a nucleus is described in terms of deuterons, tritons, trelions,[2] and helions,[3] as well as neutrons and protons not included in these clusters (spherons). In the following paragraphs I point out that many observed magnetic-moment values for the lighter nuclei can be accounted for in a simple and straightforward way by the polyspheron theory.

The polyspheron theory is compatible with the shell model,[4] and may be considered to be an extension of it. The polyspheron structures described below are in general equivalent to hybrids of two or more shell-model structures, usually (but not always) based on the most stable shell-model configuration. A polyspheron normal-state structure differing from a shell-model structure occurs when the correlation energy of two, three, or four nucleons in a localized $1s$ orbital provides greater stabilization energy than the spin-orbit interactions in the more diffuse shell-model orbitals.

Triton-Polyhelion Nuclei.—As the first example I discuss $_{13}Al_{14}{}^{27}$, which has spin-parity $5/2^+$ and magnetic moment[5] $\mu = +3.6414$. The shell-model discussion is straightforward: 14 neutrons give a completed $s^2p^6(d5/2)^6$ spherically symmetrical structure, and 13 protons a similar structure with a $d5/2$ proton hole. The value of the magnetic moment $\mu = Jg$ calculated by the vector-model equation[6] (Schmidt equation[7])

$$g(J) = 1/2(g_1 + g_2) + (g_1 - g_2)\frac{J_1(J_1 + 1) - J_2(J_2 + 1)}{2J(J + 1)} \tag{1}$$

for two vectors J_1 and J_2 combining to the resultant J (in this case $J_1 = L = 2$, $g_1 = 1$, $J_2 = S = 1/2$, $g_2 = 2\mu_p = 2 \times 2.7928$) is $\mu = 4.79$, in pronounced disagreement with the observed value. The calculation for a moving triton in $d5/2$ is the same except that $g_1 = 1/3$ and $g_2 = 2\mu_t = 2 \times 2.9789$; it gives $\mu = 3.65$, in excellent agreement with the experimental value.

There are two small corrections that might be made. The first is the Mitbewegung correction, calculated by consideration of the motion of two clusters (the triton, $E_1 = 1$, $M_1 = 3$, and the rest of the nucleus, $E_2 = 12$, $M = 24$) about their center of mass. If the center of charge and center of mass of each cluster coincide, the value of g_1 is $(E_1M_2/M_1 + E_2M_1/M_2)/(M_1 + M_2)$. With this correction the calculated value of μ is increased to 3.68. The other correction is the Jensen-Mayer[8] correction for spin-orbit coupling. The Zaretskii[9] expression $\pm$ $0.11A^{1/3} (2J + 1)/(2J + 2)$ for this correction (minus sign for $J = l + 1/2$, plus for $J = l - 1/2$), together with the factor $1/3$ for the charge/mass ratio of the triton, then leads to the value $\mu = 3.59$ for moving triton. With the two correc-

tions the moving-proton value is 5.06. Both of these corrections are uncertain, and I shall ignore them in the following discussion, except for the next example.

The normal state of $_3Li_4^7$ is $3/2^-$, with observed moment $+3.2563$. The calculated values are $+3.79$ for a proton in $p3/2$ and $+3.31$ for a triton in $p3/2$. The much better agreement with observation for moving triton than for moving proton is retained when the corrections for Mitbewegung and spin-orbit interaction are made ($\mu = 3.53$ for moving proton, 3.33 for moving triton).

For $_9F_{10}^{19}$, the shell model suggests $1d5/2$ or $2s1/2$ for the normal state, which is observed to have $J^P = 1/2^+$, $\mu = +2.6287$. There is a $5/2^+$ excited state close by (at 0.197 MeV), which has $\mu = +3.69$. This observed value agrees well with the value 3.65 for a triton in $d5/2$, and less well with the proton value, 4.79 (see the above discussion of Al^{27}).

For $_{11}Na_{12}^{23}$, with $J^P = 3/2^+$, the observed value of μ, $+2.2175$, agrees with that for a triton in $(d5/2)3/2$, 2.19, and not with that for a proton in $(d5/2)3/2$, 2.88. (These calculated values are $3/5$ of those for $d5/2$.) Moreover, the stability of $3/2^+$ as normal state (0.438 MeV below the $5/2$ state) is reasonably explained by the polyspheron theory. The close-packed configuration of five helions (Ne^{20}) places them at the corners of a triangular bipyramid, approximating a prolate ellipsoid. The most stable positions for a triton, out from the centers of the six triangular faces of the bipyramid, are the regions of maximum probability for $(d5/2)3/2$, the regions for $(d5/2)1/2$ being the polar caps and those for $(d5/2)5/2$ the equatorial belt.

The polyspheron theory assigns to the seven-helion nucleus $_{14}Si_{14}^{28}$ a pentagonal bipyramidal structure, approximating an oblate ellipsoid, and to $_{15}P_{16}^{31}$ the same structure plus a triton in $(d5/2)1/2$, which has its regions of maximum probability in the polar caps. The normal state is observed to be $1/2^+$, with $3/2^+$ at 1.27 MeV and $5/2^+$ at 2.23 MeV. The value of μ for the normal state is $+1.13166$, somewhat larger than for a triton in $(d5/2)1/2$, 0.729, and much smaller than for a triton in $s1/2$, 2.979. Presumably a mixed-configuration structure (82% d, 18% s) is to be assigned to this nucleus. The alternative simple explanation of the observed value, that there is a moving proton in $(d5/2)1/2$, has to be rejected, the calculated value of μ being only $+0.959$. A mixed-configuration structure with moving proton would account for the observed moment.

The normal state of Sc^{43} is $7/2^-$, with $\mu = 4.52$. The calculated values are 3.98 for a moving $f7/2$ triton and 5.79 for a moving $f7/2$ proton. The observed value indicates that the structure is an intermediate one.

The other nuclei with $Z = N - 1$, N even, and with known values of the magnetic moment are B^{11}, N^{15}, Cl^{35}, and K^{39}. For each of these nuclei the neutrons constitute a completed shell or subshell, and would be expected not to contribute to the angular momentum or magnetic moment. The polyspheron description of these nuclei would attribute the orbital angular momentum to the resonance of the proton hole from one spheron to the next one, and would lead to the same calculated values of the magnetic moment as the shell model.

Discussion and Summary.—In Figure 1, the heavy horizontal lines connected by the dashed lines (Schmidt lines) represent the moving-proton values given by equation (1) with $g_l = 1$ and g_s equal to twice the proton spin moment. The upper dashed line is the Schmidt line for $J = l + 1/2$ (sequence $s1/2$, $p1/2$, $d1/2$,
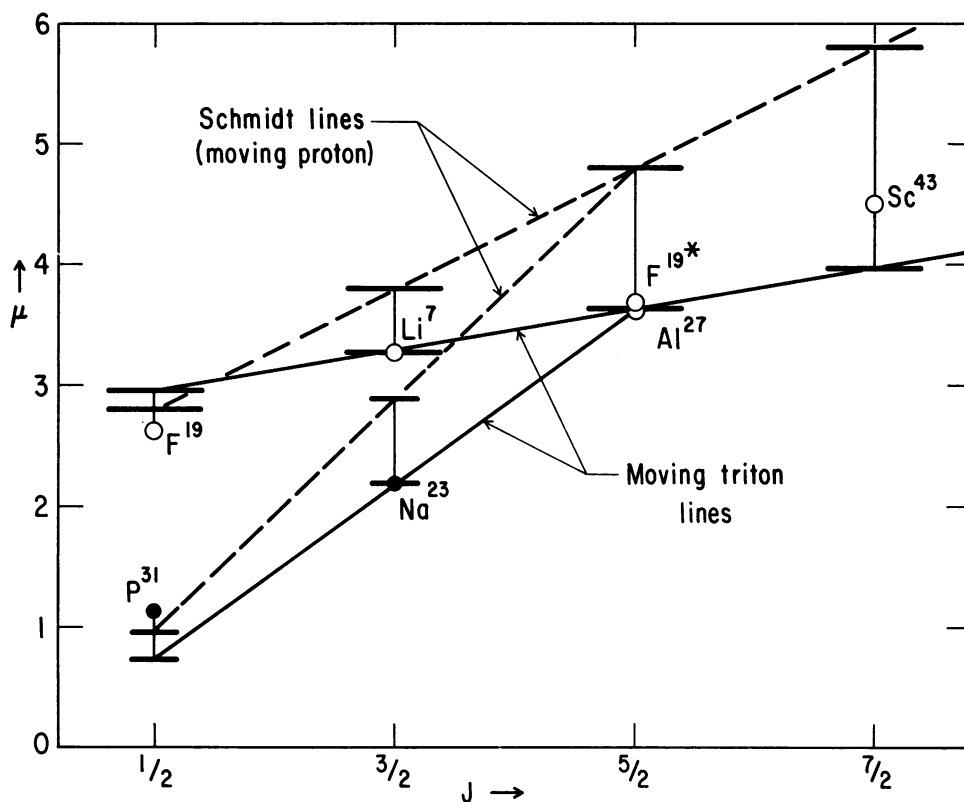
FIG. 1.—Magnetic moments of triton-polyhelion nuclei.

$f1/2$). The lower Schmidt line connects the calculated values for $(d5/2)1/2$, $(d5/2)3/2$, and $(d5/2)5/2$. The solid lines connect the corresponding calculated moving-triton values (eq. (1), $g_l = 1/3$, g_s twice the triton spin moment). Four of the five open-circle experimental points are seen to lie closer to the moving-triton line than to the Schmidt line. The point for Na^{23} [$(d5/2)3/2$, solid circle] also lies closer to the moving-triton value than to the Schmidt value. The magnetic-moment values of these light nuclei are accordingly compatible with the poly-spheron (triton-polyhelion) description of their structure. The observed value for Sc^{43} suggests that the moment of this larger nucleus results from contributions of about 70 per cent by a moving triton and 30 per cent by a moving proton, but this conclusion might be changed by a more refined calculation, with consideration of Mitbewegung, spin-orbit interaction, and polarization.[10]

The results of this study indicate that a zero-order wave function representing a moving triton may constitute a better starting point for a quantum-mechanical discussion of the structure of a light nucleus than a zero-order function representing a moving proton.

[1] Pauling, L., Science, 150, 297 (1965).

[2] Pauling, L., *Rev. Roumaine Phys.*, **11**, 825 (1966).

[3] Pauling, L., *Nature*, **201**, 61 (1964).

[4] Goeppert-Mayer, M., and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (New York: Wiley, 1955).

[5] Values of magnetic moments are from Lindgren, I., in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, ed. K. Siegbahn (Amsterdam: North Holland Publishing Co., 1965).

[6] See, for example, Pauling, L., and S. Goudsmit, *The Structure of Line Spectra* (New York: McGraw-Hill Book Co., 1930).

[7] Schmidt, T., *Z. Physik*, **106**, 358 (1937).

[8] Jensen, J. H. D., and M. Goeppert-Mayer, *Phys. Rev.*, **85**, 1040 (1952).

[9] Zaretskii, D. F., *Soviet Phys.-JETP*, **9**, 612 (1959).

[10] Blomqvist, J., N. Freed, and H. O. Zetterström, *Phys. Letters*, **18**, 47 (1965); Bhaduri, R. K., and P. van Leuven, *Phys. Letters*, **20**, 182 (1966).